Mesoscopic Modeling of Vapor Transport in Wood in Tangential and Radial Direction

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ABSTRACT

Wood is traditionally modeled as an orthotropic material, with direction dependent transport properties, in this way homogenizing the specific structure of wood: a cellular material with growth rings, consisting of early wood and late wood layers. In this paper we present a mesoscopic model for vapor transport in wood, where earlywood and latewood are modeled as distinct materials. The sorption and vapor transport properties for early- and latewood are determined using a unit cell model with constant cell geometry. These properties are used in two-dimensional calculations of vapor transport in a piece of wood consisting of several growth rings. The mesoscopic results are compared to measurement results and to macroscopic results using homogenized orthotropic transport properties. For vapor transport in radial and tangential direction the difference between meso- and macro modeling of wood is negligible, justifying the vapor transport modeling with homogenized orthotropic transport properties.

INTRODUCTION

In building physics wood is mostly modeled as a homogeneous isotropic material. Orthotropy refers to different transport properties in longitudinal, radial and tangential direction. In reality wood is made of growth rings, consisting of late wood layers with thick walled cells and earlywood with cells with small wall thickness and wide lumen. A two-scale model to determine the homogenized transport properties is formulated by Siau 1995. At the microscale, a unit cell approach is used to determine the early- and latewood properties. These properties are further upscaled using a representative elementary volume (REV) consisting of plane layers of early- and latewood. In Siau's approach the specific curved shape of the growth rings is neglected. The question arises if this two-scale approach is valid for vapor transport simulations in wood.

In this paper, we present a mesoscopic model for wood explicitly modeling the growth ring structure in two dimensions. First, we present measurements results of vapor transport properties of wood. Then the unit cell method is used to determine the vapor transport properties of earlywood and latewood. In a third part, the mesoscopic approach is presented and compared to the REV method of Siau. Finally, the dynamic sorption behavior of wood is modeled using the mesoscopic approach. The mesoscopic results are compared to measurements and to macroscopic results using homogenized vapor transport properties.

HYGROSCOPIC BEHAVIOR OF WOOD

Theory of Water Vapor Transport

Isothermal water vapor transport in orthotropic materials is described by

$$\frac{\partial w}{\partial t} = \nabla\delta\nabla p_v \qquad (1)$$

with w the moisture content [kg/m^3], t time [s], the water vapor permeability tensor [s] and p_v the water vapor pressure [Pa]. The vapor permeability tensor is given by

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$$\delta = \begin{bmatrix} \delta_L & 0 & 0 \\ 0 & \delta_R & 0 \\ 0 & 0 & \delta_T \end{bmatrix} = \delta_a \begin{bmatrix} 1/\mu_L & 0 & 0 \\ 0 & 1/\mu_R & 0 \\ 0 & 0 & 1/\mu_T \end{bmatrix} \tag{2}$$

where the subscript L, R, T refer to respectively longitudinal, radial and tangential direction, δ_a is the water vapor permeability of air [s] and μ is the water vapor resistance factor [-].

Further derivation gives

$$\frac{\partial w}{\partial \phi} \cdot \frac{\partial \phi}{\partial t} = \xi \cdot \frac{\partial \phi}{\partial t} = \nabla \delta \cdot p_{vsat} \nabla \phi \tag{3}$$

is the moisture capacity [kg/m^3], ϕ the relative humidity (RH) [-],and p_{vsat} the saturation water vapor pressure [Pa]. The function describing $w(\phi)$ is called the sorption isotherm.

Material

The wood tested in this paper is spruce *(Picea abies)* from Bavaria, all cut from the same log. The average dry density is 402 kg/m^3, but varies between 364 kg/m^3 and 454 kg/m^3 depending on the location in the cross section. On average, the volume ratio of latewood to earlywood determined from microscopic analysis is 0.2.

Sorption Isotherm

The main adsorption isotherm of wood is measured by conditioning samples in a dessicator over saturated salt solutions at a controlled temperature of 23°C. The samples were initially dried at 50°C and 10% RH. The adsorption isotherm is described by

$$w(\phi) = w_{max} S(\phi) = (1 + (a_1 \cdot \ln\phi)^n)^{-m} \tag{4}$$

with S the degree of saturation, w_{max} the moisture content at 97% relative humidity (*RH*). Equation (4) resembles the function as proposed by van Genuchten 1980, but modified by introduction of the relative humidity as potential instead of the capillary pressure. Kelvin's law is used for the transition from capillary pressure to relative humidity. We assume that, at 97% RH, the cell walls are saturated and further moisture storage will only occur by filling the cell lumen. The parameters a_1, n and m are fitted to the measurement data: a_1 = -3.870, n = 1.353 and m = 0.732, w_{max} = 87.6 kg/m^3. The measurement data and fitted curve are shown in Figure 1.

Water Vapor Permeability

The water vapor permeability was measured using the dry cup/wet cup method at three different relative humidity ranges. The samples were first dried at 40°C and 3% RH in order to be in main adsorption mode. It is assumed that the water vapor resistance factor is constant in a certain RH range [ϕ_1, ϕ_2], where ϕ_1 and ϕ_2 are the RH's on each side of the sample. The measured water vapor resistance factors in radial and tangential directions are given in Table 1. We observe that the water vapor resistance factors depend highly on the relative humidity, and that they are comparable in tangential and radial directions.

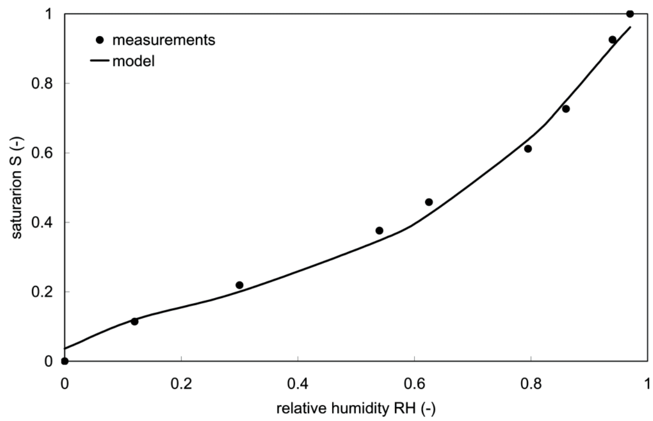


Figure 1 *Main adsorption isotherm* S(ϕ)*, where* S *is the degree of saturation. The dots represent the measurement data and the solid line represents the fitted curve using Equaion 4.*

DETERMINATION OF VAPOR TRANSPORT PROPERTIES OF EARLY AND LATEWOOD

Density and Sorption Isotherm

We may assume that the cell walls of earlywood and latewood are composed of the same material, i.e. early- and latewood only differ in the cell geometry and cell wall thickness. The average cell dimensions are obtained from scanning electron microscopy (SEM) of wood (Figure 2). The dimensions are given in Table 2, with L the length of the cell, t the thickness of the cell wall. Assuming a bulk density of the cell wall of 1500 kg/m^3 Kollmann and Côté 1968), the densities of earlywood and latewood can be determined (Table 3).

The sorption isotherm of the total wood is given by equation (4). Since the cell walls of early- and latewood are composed of the same material, the sorption isotherms of early- and latewood will only differ in the maximal moisture content. The sorption isotherms of earlywood and latewood are then given by

$$w_e = w_{max,e} S(\phi) \quad , \quad w_1 = w_{max,l} S(\phi) \tag{5}$$

The maximum moisture content for earlywood $w_{max,e}$ and latewood $w_{max,l}$ can be determined according due their density, ρ_e and ρ_l, or

$$w_{max,e} = w_{max} \cdot \frac{\rho_e}{\rho} \quad , \quad w_{max,l} = w_{max} \cdot \frac{\rho_l}{\rho} \tag{6}$$

The maximum moisture content for early- and latewood are given in Table 3.

Table 1. Measured and Simulated Vapor Resistance Factors for Spruce in Radial and Tangential Directions.

	μ		
	30% RH	**62.5% RH**	**71.5% RH**
Measured			
Radial	273	72	29
Tangential	341	68	26
REV Model			
Radial	238	60	26
Tangential	341	75	28
Two-Dimensional Model			
Radial	414	91	34
Tangential	288	64	24

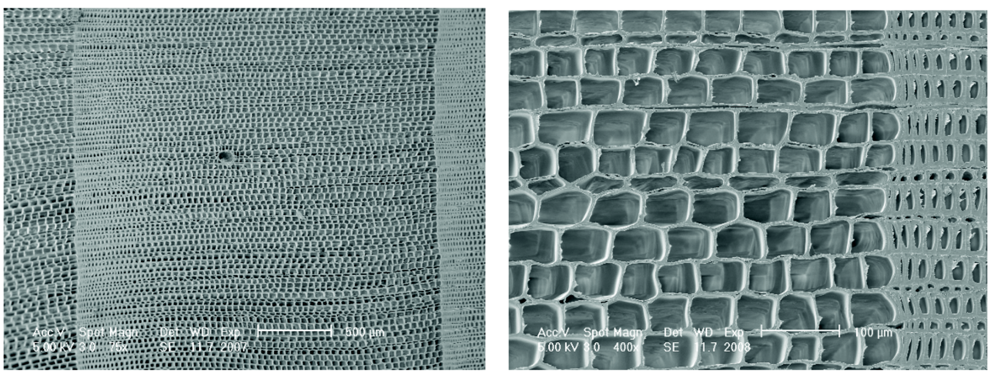


Figure 2 *SEM images used to determine the cell geometry of the wood, as well as the growth ring width and the earlywood/latewood ratio.*

Table 2. Cell Geometries Used in the Microscale Model.

	Earlywood	**Latewood**	**Rays**
L_L (μm)	2800	2800	700
t_L (μm)	3	3	3
L_R (μm)	56	22	10
t_R (μm)	3	5	2
L_T (μm)	40	40	10
t_T (μm)	2	5	2

Vapor Permeability

The vapor permeability of early- and latewood are determined using the unit cell model as proposed by Siau 1995. The unit cell has dimensions L_L, L_R, L_T and wall thicknesses t_L, t_R, t_T. Assuming symmetry, only one quarter of a cell has to be considered (Figure 3). The vapor permeability of the unit cell is determined using the parallel-serial resistance model of the quarter of the unit cell. Since the cell walls are made of the same material, the vapor permeabilities of the wall in radial and tangential direction and for latewood and earlywood are equal. The vapor permeability of the wall depends on the moisture content related to a given RH. The vapor permeability of the wall was set to 927 at 30% RH, 203 at 62.5% RH and 74 at 71.5% RH. These values were determined by inverse identification, fitting the water vapor resistance factors in radial and tangential direction as given by the REV model (see further) to the measured values. Knowing the vapor perme-

Table 3. Earlywood and Latewood Densities as Well as the Maximum Sorptive Moisture Contents.

	Density ρ kg/m^3	**w_{max} kg/m^3**
Earlywood	297	65
Latewood	888	194

Table 4. Water Vapor Resistance Factors for Earlywood and Latewood Determined from the Unit Cell Model.

		μ	
	30% RH	**62.5% RH**	**71.5% RH**
Earlywood			
Tangential	258	57	22
Radial	274	61	23
Latewood			
Tangential	881	194	71
Radial	1221	268	98

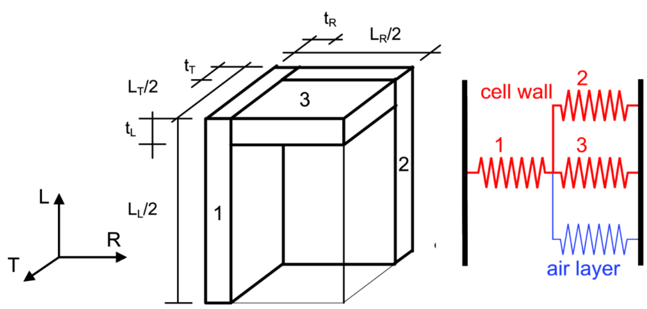


Figure 3 *One edge of the cell model is shown with the definition of the cell dimensions. The equivalent resistance model for vapor transport in radial direction is also shown on the right-hand side.*

ability of air, the vapor permeability in radial and tangential direction can be determined (see Table 4).

We observe that earlywood has a lower vapor resistance factor (higher vapor permeability) compared to latewood. This can be explained by the difference in wall thickness. The decrease of the vapor resistance factor (increase of the water vapor permeability) at high relative humidity is explained by the fact that microscopic liquid water transport in filled pores enhances the apparent water vapor transport. This means that the apparent water vapor diffusion coefficient depends on the degree saturation. We model the water vapor permeability as

$$\delta(S) = \frac{\delta_a}{\mu(S)} \text{ with } \mu(S) = \frac{1}{a + b \cdot e^{cS}} \tag{7}$$

Figure 4 gives the water vapor resistance factors as a function of degree of saturation for latewood and earlywood. The parameters a, b and c are given in Table 5.

MESOSCOPIC MODELING OF WOOD

In this section we present two models for predicting the macroscopic vapor permeability of wood. Both models are based on the modeling of wood on the mesoscale, consisting of layers of early- and latewood. The vapor transport properties of late and earlywood determined in section 3 are used.

REV Model

Following Siau 1995, the existence of a REV (representative elementary volume) is assumed consisting of two parallel plane layers, one of early- and one of latewood, and a ray layer normal to the other layers (Figure 5). The assumption of the existence of a REV inherently implies that this unit is representative for the complete wood. The average fraction of earlywood f_e is determined based on SEM images. The fraction f_e is taken constant and equal to 0.833 (the volume ratio of latewood to earlywood is 0.2). The ratio n of the volume of ray-layer to the total volume is taken equal to 0.05. The vapor permeability of wood in radial and tangential direction is determined based on parallel – serial formulas. The results are given in Table 1. It is noted that the water vapor permeability of the wall is used as fitting parameter. We observe that the REV model gives a good agreement with the measurements.

Two-Dimensional Mesoscopic Model

In the mesoscopic approach, we model the vapor transport in a small block of wood of 10 mm x 10 mm assuming equally spaced growth rings (Figure 6b). The ring width is based on average ring dimensions of 3 mm. Rays are not considered. The stationary water vapor transport of wood is simulated using a two-dimensional finite element model for heat and moisture transfer in porous materials (Janssen et al. 2005). The finite element mesh in given in Figure 6c. We assume isothermal conditions (23°C). An uniform moisture

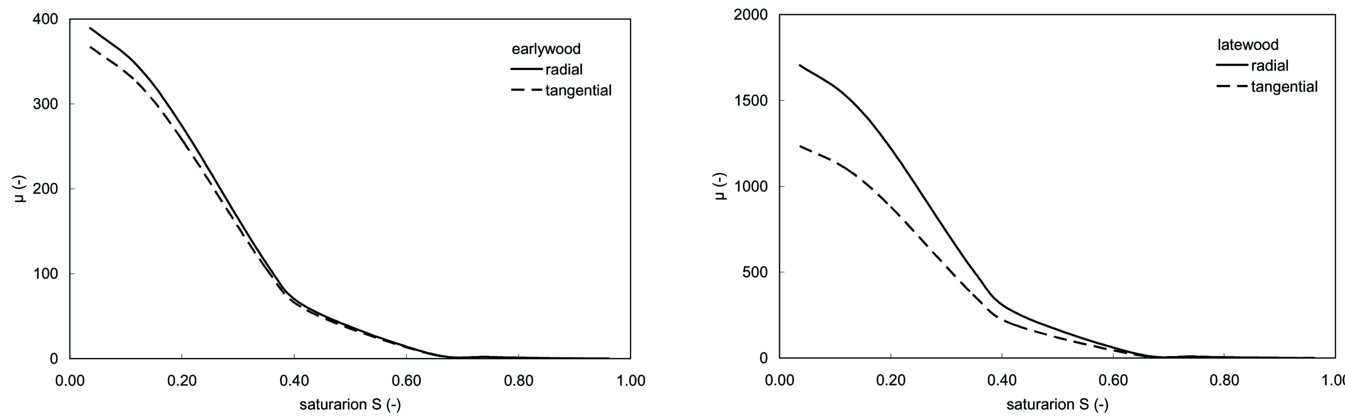


Figure 4 *Obtained curves of the water vapor resistance factor for earlywood (left) and latewood (right).*

Table 5. Parameters for the Water Vapor Resistance Factors.

	Earlywood		Latewood	
	Radial	**Tangential**	**Radial**	**Tangential**
a	$2.34 \cdot 10^{-3}$	$2.48 \cdot 10^{-3}$	$5.40 \cdot 10^{-4}$	$7.45 \cdot 10^{-4}$
b	$1.54 \cdot 10^{-4}$	$1.65 \cdot 10^{-4}$	$3.11 \cdot 10^{-5}$	$4.40 \cdot 10^{-5}$
c	$1.07 \cdot 10^{1}$	$1.07 \cdot 10^{1}$	$1.10 \cdot 10^{1}$	$1.09 \cdot 10^{1}$

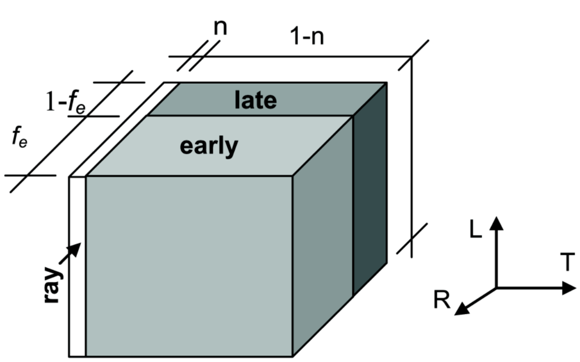


Figure 5 *Growth ring model to scale latewood and earlywood vapor permeabilities from the cell model up to macro level.*

content is assumed in the material corresponding to a given relative humidity. Imposing a small difference in RH over the specimen the stationary vapor flow through the material is calculated. From the calculated vapor flow and RH difference the effective vapor permeability or vapor resistance factor of the specimen is determined. The results are given in Table 1. Generally a good agreement with the measurement data is found. Only at low relative humidity a higher vapor resistance factor is obtained. This is probably due to the fact that the influence of the wood rays is not taken into account in the 2D mesoscopic model.

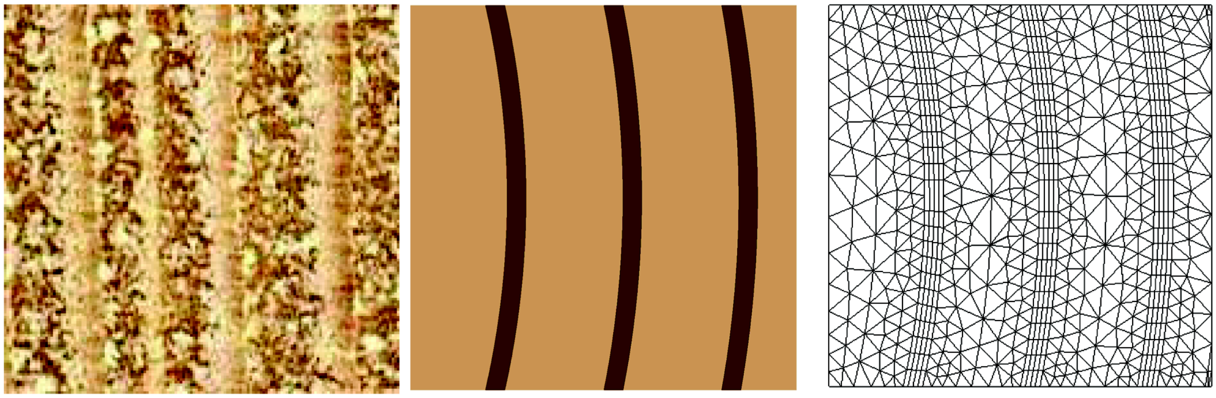
We conclude that both the REV model and the mesoscopic model give comparable results and that the influence of the curvature of the rings is limited.

DYNAMIC HYGROSCOPIC LOADING OF WOOD

The dynamic sorption behavior of wood is measured using the experimental set-up of the Building Physics and Systems Unit at T.U. Eindhoven. The thickness of the specimen is 10 mm. The samples are made vapor tight with wax at bottom and lateral sides so that a one-dimensional moisture uptake is achieved. The specimen are first dried above silica gel and then conditioned at 23°C and 54% RH until equilibrium. In this way a pure adsorption behavior is obtained. The specimens are then exposed to a step change in RH to 79.5% during 14 days. The weight increase of the sample is measured every 600 s. The test is performed for the radial and tangential direction of wood. Figure 7 gives the measurement data for the radial direction.

The adsorption behavior of wood is modeled using two approaches. The first approach is the classical approach where the vapor transport is modeled on the macroscale using direction dependent vapor permeability for wood. The simulation is one-dimensional. The model is referred to as macroscopic model. Homogenized vapor transport properties as determined by the REV model are used.

In a second approach we model the behavior of wood on the mesoscale including the presence of layers of latewood and earlywood. Different vapor permeabilities, dependent on



(a) (b) (c)

Figure 6 *(a) Growth ring structure of wood, (b) idealized mesostructure (light brown: earlywood; dark brown: latewood), and (c) finite element mesh used in the simulations.*

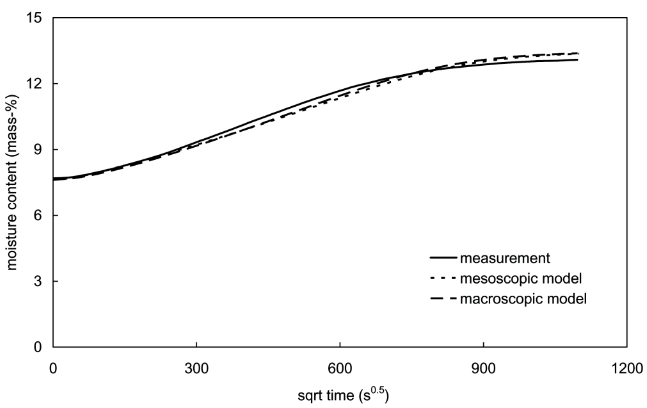


Figure 7 *Comparison of modeled moisture content to measured moisture content during wetting period in the step change test.*

direction, and different sorption isotherms for latewood and earlywood are used, as determined in section 3 by the unit cell method. The simulation is two-dimensional. This approach is referred to as mesoscopic model. The sorption behavior of a small block of wood of 10 mm x 10 mm is modeled assuming equally spaced growth rings (see Figure 8a). The ring width is based on average ring dimensions of 3 mm.

The surface transfer coefficient in both approaches is fitted to the measurement data and assumed to be constant during the whole test period. The surface coefficient equals $4.0 \cdot 10^{-9}$ s/m. The non-stationary water vapor transport in both approaches is solved using the finite element model for heat and moisture transfer in porous materials (Janssen et al. 2005). This model is based on a finite element spatial and an implicit time discretisation. Figure 8b gives the finite element mesh for the mesoscopic approach.

Figure 7 gives the moisture uptake for the mesoscopic and macroscopic model. A good agreement between the measurements and the macroscopic and mesoscopic models is obtained. The results of macroscopic and mesoscopic models do not differ substantially, which means that the macroscopic approach using homogenized orthotropic transport properties is valid for vapor transport in wood. Figure 8 gives the wetting patterns in the wood during the first 32 hours as obtained from the mesoscopic model. We clearly see that the wetting pattern reflects the different vapor transport properties of latewood and earlywood. However, the moisture front builds almost uniformly up, justifying the validity of a macroscopic approach.

DISCUSSION AND CONCLUSION

In this paper we presented a mesoscopic and macroscopic approach for vapor transport in wood. The vapor transport properties for latewood and earlywood are determined using a unit cell model following Siau 1995. The average dimensions of the wood cell and the layer thickness of earlywood and latewood are determined from SEM images. To determine the macroscopic vapor transport properties, a REV (representative element volume) model as proposed by Siau 1995 and a mesoscopic model are considered. In the REV model, the curved geometry of the wood layers is neglected. In the mesoscopic model, the geometry of the layers is realistically modeled, but the presence of rays is not taken into account. A good agreement was obtained between the REV and mesoscopic model and between predicted and measured vapor permeabilities at three relative humidities. Finally the dynamic adsorption behavior of wood was modeled using macroscopic and mesoscopic finite element simulations. The mesocopic simulations revealed that the moisture distribution in the wood during wetting is rather uniform, justifying the validity of the macroscopic approach using homogenized transport properties. Therefore it can be concluded that it is valid to use macroscopic direction dependent vapor transport properties for modeling sorption behavior of wood.

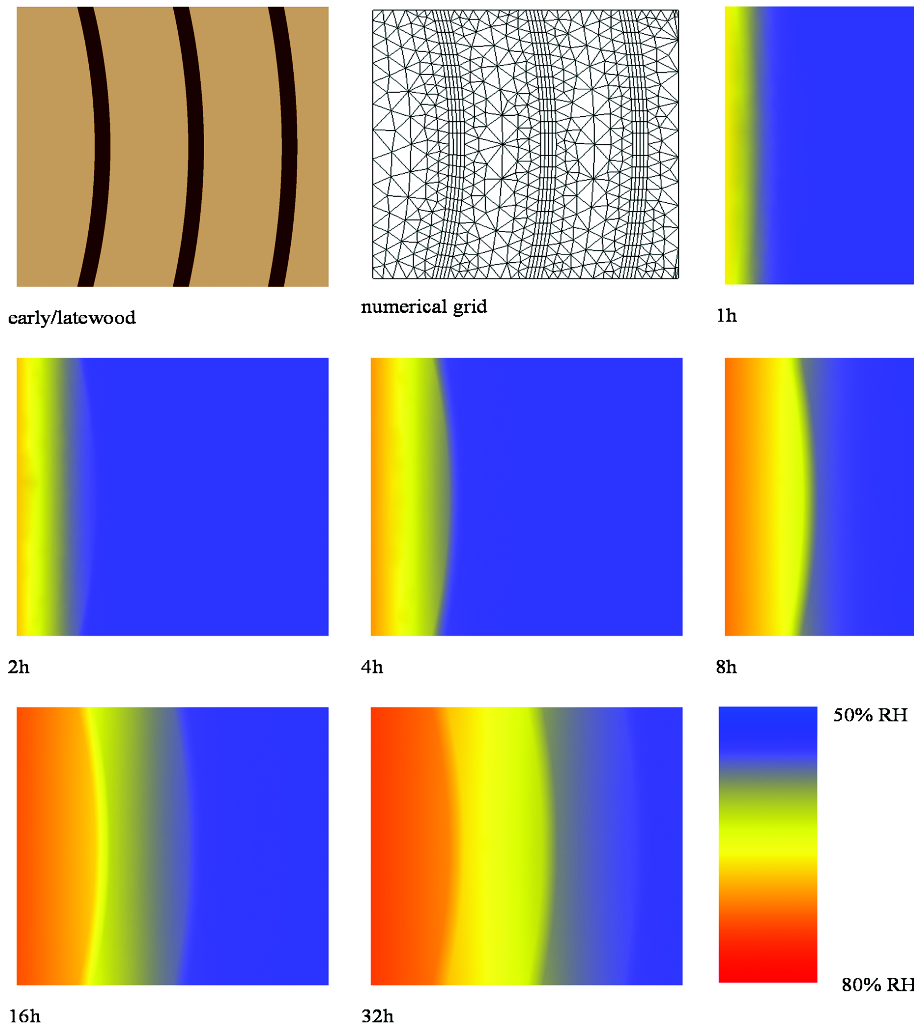


Figure 8 *Moisture distribution in the wood during the wetting period of the step change test.*

In future research, the mesoscopic model will be further explored. The following influencing factors or phenomena will be analyzed: non-equally spaced earlywood and latewood layers, cell geometry differences, the size of the wood specimen, orientation of the wood layers, the presence of anomalies, defects and cracks, etc. In a next step the swelling of the material and the change of the cell dimension will be taken into account. The approach will be further extended to liquid transport. In liquid transport the pits between the different lumens will play an important role, where for vapor transport the presence of pits has a negligible influence (see Siau 1995).

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